Common Sense Reasoning About Complex Actions

Abstract

Reasoning about complex actions in partially observed and dynamic worlds requires imposing restrictions on the behavior of the external world during their occurrences. In this paper, we use the first-order ontology of the Process Specification Language to characterize a class of models of the ontology that captures the intuitive scenarios where interference from the external world is minimized. We provide a circumscription based preference policy to allow reasoning within these models.

Introduction

Complex actions consist of primitive actions and other complex actions. A definition for a complex action involves a specification of the actions that occur as part of its occurrence along with the constraints on their occurrences. A complex action is intended to capture a complex behavior. At the concrete level, the behavior is a sequence of actions to be performed in a given situation. At the semantic level, the behavior is a manipulation of the world, such that, performing the behavior moves the world from one state to another where the new state is ensured to posses some desired properties that the earlier state lacked. The designing process of a complex action typically motivated by a need for a behavior that manipulates the world in a certain way. One then figures out what sequences of actions available in the domain would achieve this effect in various possible situations. Finally, a compact representation of these sequences are encoded in the language of the underlying framework which is a specification of the complex action.

Reasoning with complex actions entails going in the opposite direction. Given a complex action specification, we would like to find out, at the concrete level, say, the actions that must occur whenever the complex activity occurs, the actions that can possibly occur, the actions that always occur in some order and so forth, and, at the semantic level, we would like to know the state parameters that are affected by an occurrence of the complex action, the way they are affected, the state parameters that stay untouched if the action occurs under certain conditions and so forth.

In a logical framework all these properties of the complex actions are characterized by their models. Therefore, reasoning about them reduces to the decision problem of what is entailed by and what is consistent with these models. In order to be able to represent these decision problems as first order sentences, the complex actions and their occurrences must be first class objects in the language. The underlying framework we use in this paper, the Process Specification Language (PSL)((Grüninger 2003), (Grüninger and Menzel 2003)), provides the expressive power to reason about the complex actions within first order logic.

Reasoning about complex actions in a partially observed dynamic world, however, requires going beyond standard first-order entailment based reasoning. Suppose a complex action *register-to-conference* involves the actions booking a flight, booking a hotel and registering to the conference. The hotel check-in date must be the same day as the flight, so the flight must be booked before the hotel. Suppose also that flight rate must not exceed 500CAD and the hotel rate must not exceed 100CAD/day for three days and registering to the conference costs 400CAD. Based on this description we would like to conclude that whenever this action is performed, a flight is booked before a hotel, no other flight is booked after the hotel is booked, the client is registered to the conference, and the clients credit card account balance has not been increased by more than 1200CAD compared to what it was before performing the action, among other things.

Unfortunately, not all of these conclusions logically follow from the description. For example, another action, say, reservation in a holiday resort for the same client can be performed concurrently with the action which would probably violate the last conclusion. The description ensures that a flight is booked before a hotel but another flight booking for the client after the hotel is booked cannot be ruled out. The client can even be unregistered from the conference by another action before the action is completed.

In general, very few conclusions can be made about a complex action's occurrence, if any, based on its description if other actions that are not specified in the description of the action can also occur. In fact, the description does not even rule out other actions' occurrences to be conceptually part of the registerToConference action. In other words the description does not say that the three actions in the description are the only actions that must occur to perform the registerToConference action, nor does it say that they must occur only once. A some sort of closed world assumption about the occurrences of the unspecified actions must be made to be able to extract useful information about a complex action from its description. The closed world assumption cannot be simply the assumption that no other action occurs, since some of the unspecified actions may be implicitly required to occur. In the example above the client's credit card account must be debited for the subactions of the activity to be performed.

This paper will provide characterizations of the *closed* models of complex action descriptions within which reasoning yields the intended conclusions. We will also present the circumscription policies that correspond to those closed models.

PSL Overview

Process Specification Language (PSL) is a standardized (ISO 18629) upper ontology for the exchange of information among manufacturing processes. The ontology is a set of first-order logic theories. A subset of these theories is called PSL-Outer Core and it contains the theories that axiomatize the intuitions and concepts about manufacturing processes. The rest of the ontology consists of definitional extensions that classify the models of the PSL Outer Core with respect to the invariants in the models. Intuitively, the definitional extensions identify classes of activities and objects representable with the PSL Outer Core. The work presented here applies to all the models of the PSL Outer Core, i.e. all classes of activities within the PSL Ontology, therefore we will not further discuss the definitional extensions.

The PSL Ontology contains four distinct classes of elements; *activities*, *occurrences*, *timepoints* and *objects*. Activities correspond to reusable behaviors. An activity's occurrence that start and end at particular timepoints is represented with an occurrence. Activities can have multiple occurrences or no occurrences at all. However every occurrence is associated with a distinct activity. The relationship between the activities and occurrences is specified with the *occ_of(o,a)* relation which says that o is an occurrence of the activity a. Timepoints form an infinite linearly ordered set. Everything else in a domain is an object.

Activities can be primitive, atomic or complex. For every set of primitive activities there exists an atomic activity whose occurrence represents concurrent occurrences of every activity in the set. Therefore in PSL, concurrent occurrences of multiple activities is a distinct phenomena that their individual occurrences. Effects and preconditions of participating activities are not inherited by the concurrent activity. Participating activities are subactivities of the concurrent activity they form. Every primitive activity is also an atomic activity and hence, subactivity of itself. Complex activities also have subactivities. Their subactivities can be atomic or other complex activities.

The PSL Ontology also axiomatizes an occurrence tree structure. For every possible sequence of atomic activity occurrences, there is a branch in the occurrence tree that corresponds to the sequence. Occurrence trees are isomorphic to situation trees of the situation calculus formalism(McCarthy and Hayes 1969)(Reiter 2001) with one difference that occurrence trees do not have an initial situation. For every atomic activity there is an occurrence of that activity that is the root of an occurrence tree. Therefore every model of a domain theory extending the PSL Ontology contains as many occurrence trees as there are atomic activities in the domain.

The state information in the PSL Ontology is represented with the properties called fluents. Fluents are reified in PSL, in other words they are objects in the language. Fluents can only be changed by the occurrence of activities. There are two relations that link the fluents and the occurrences; *prior(f,o)* is true if the fluent, f, is true before the occurrence, o, and *holds(f,o)* is true if f is is true after o.

The occurrence trees represent combinatorically possible sequences of activities, however not every sequence will be possible in a domain. A contiguous subtree of the occurrence tree called *legal occurrence tree* contains only those occurrences that are possible in a domain. The *precedes*(o_1, o_2) is the ordering relation on the legal occurrence tree, specifying that o_1 comes before o_2 on a branch of the legal occurrence tree.

An occurrence of a complex activity corresponds to a sequence of atomic subactivity occurrences starting with a root occurrence. In general, there are many sequences of atomic subactivity occurrences with the same root occurrence consistent with a process description. The structure that represents the set of these sequences is called an *activity tree* in PSL. Activity trees are embedded into the legal occurrence tree (not necessarily contiguously). Each branch of an activity tree is an occurrence of the complex activity it is associated with. Consider the complex activity presented in the introduction section. A process description for that activity can be given as follows:

$$\begin{aligned}
&\forall o. occ_of(o, \text{registerToConference}) \supset \\
&\quad \exists s_1, s_2, s_3. occ_of(s_1, \text{bookFlight}) \wedge \\
&\qquad occ_of(s_2, \text{bookHotel}) \wedge occ_of(s_3, \text{register}) \wedge \\
&\qquad subact_occ(s_1, o) \wedge subact_occ(s_2, o) \wedge \\
&\qquad subact_occ(s_3, o) \wedge \\
&\qquad min_prec(s_1, s_2, \text{registerToConference})
\end{aligned} \tag{1}$$

The *subact_occ* relation holds between the atomic

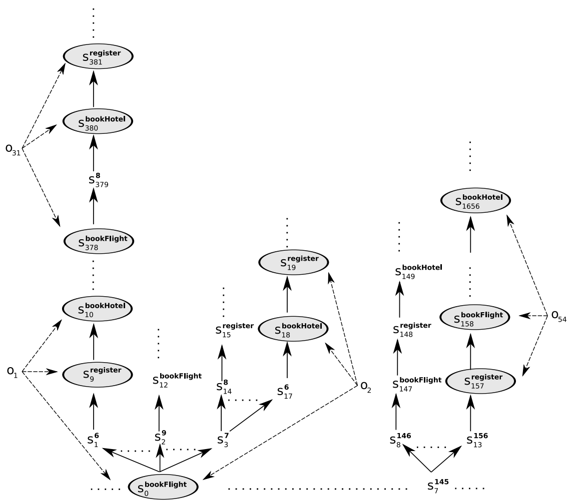

Figure 1: A model of the registerToConference complex activity description.

and complex occurrences. It specifies the atomic occurrences that are subactivity occurrences of a complex occurrence. The *min_prec* relation defines the partial ordering of the activity tree. For example *min_prec(s_1,s_2,registerToConference)* specifies that s_1 and s_2 are atomic subactivity occurrences of an occurrence of the registerToConference activity, i.e. they are on the same branch of an activity tree of the registerToConference activity, and s_1 is earlier than s_2. A possible model of the activity is shown in Figure 1.

The underlying trees are the legal occurrence trees. Branches of these trees are infinitely long and there are also other legal occurrence trees rooted on other atomic activity occurrences that are not explicit in the figure. Circled occurrences represent the subactivity occurrences of the occurrences of the complex activity. The dashed arrows visualize the subact_occ relation. The occurrences o_1, o_2, o_{31} and o_{54} are some of the occurrences of the registerToConference activity in the model. The activity trees for the complex occurrences in the figure is shown without the underlying occurrence trees in Figure 2.

Minimization of Activity Trees

A natural way to describe a complex activity is specifying the occurrences of its subactivities that must exist as part of its occurrence along with the constraints among these occurrences. It is typically "understood" from such a description that the subactivity occurrences that are not mentioned in the description are not elements of an occurrence of the complex activity. In other words, subactivity occurrences that are mentioned in the description are the only intended ones to exist.

Consider a complex activity, δ, whose occurrences consist of an occurrence of its subactivity, a_1, followed

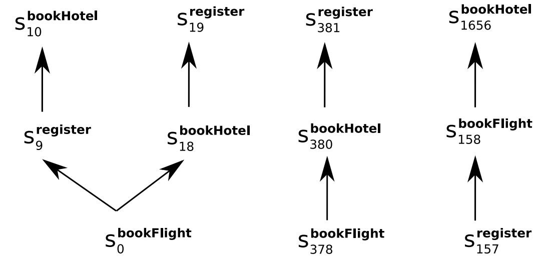

Figure 2: Activity trees of the registerToConference activity.

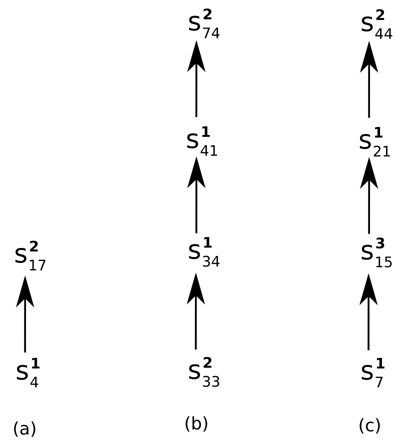

Figure 3: Possible models of the process description for the activity δ.

by an occurrence of its other subactivity, a_2. A tempting process description for the activity would be:

$$subactivity(a_1, \delta) \wedge subactivity(a_2, \delta) \tag{2}$$

$$\begin{aligned} &\forall o.occ\_of(o, \delta) \supset \\ &\quad \exists s_1, s_2.occ\_of(s_1, a_1) \wedge occ\_of(s_2, a_2) \wedge \\ &\qquad subact\_occ(s_1, o) \wedge subact\_occ(s_2, o) \wedge \\ &\qquad min\_prec(s_1, s_2, \delta) \end{aligned} \tag{3}$$

Intuitively, we would like to conclude that an occurrence of a_1 followed by an occurrence of a_2 is the only way the activity δ can occur. Figure 3.a shows one such occurrence which can be a branch of an activity tree of δ or an activity tree itself.

However, the occurrence description does not rule out multiple occurrences of the activities a_1 and a_2. For instance, an occurrence of δ that consists of subactivity occurrences of a_2, a_1, a_1 and a_2 as shown in Figure 3.b is consistent with the description. It is also consistent that δ has other subactivities. In fact, any activity that is not a super-activity of δ can be a subactivity of δ and can occur as part of an occurrence of δ as long as the occurrence contains occurrences of a_1 and a_2 in the specified order. Figure 3.c shows such an occurrence of δ where a subactivity occurrence of an activity that is not mentioned in the description, a_3, follows a root occurrence of a_1.

Obviously, the activities that are subactivities of δ can be restricted with an axiom like the following:

$$\forall a.subactivity(a, \delta) \supset a = a_1 \wedge a = a_2 \qquad (4)$$

With this axiom, the occurrence in Figure 3.c is ruled out to be an occurrence of δ. In general, however, subactivities can be concurrent activities or complex activities themselves. When this is the case, closing the extension of the subactivity relation to only include the activities that appear in the process description may not be possible. Suppose, a_1 is a concurrent activity with the subactivities a_3 and a_4. In this case, the closure of the subactivity relation must also include the activities a_3 and a_4 since the subactivity relation is transitive. Even if the activities a_1 and a_2 are primitive, the closure of the subactivity relation does not eliminate the models of the process description that contain arbitrarily large number of subactivity occurrences of them.

Clearly, the description is not complete without the addition of *closure* constraints that rule out unintended occurrences to be part of an occurrence of δ. Note that this effect cannot be achieved by turning the formula into an equivalence. The missing only if direction is entailed by the PSL Ontology. If s_1 and s_2 are subactivity occurrences of o, and they are on a branch of the activity tree of δ then o is an occurrence of δ. The solution is either an explicit statement that rules out multiple occurrences of a_1 and a_2 as well as occurrences of other activities or inclusion of some sort of a closed world assumption with respect to the occurrences descriptions in the theory.

It is important to observe that the need to add closure constraints to the complex activity descriptions is not an issue specific to the PSL Ontology. Any framework rich enough to represent complex activities and their occurrences as objects in the language and allows first order descriptions for them, will require the descriptions to be closed in the sense described above to be able to reason correctly with them. A definition of a complex activity that specifies the subactivity occurrences that are involved in performing the activity, must provide the necessary and sufficient conditions to be satisfied in order for an event to be considered an occurrence of the activity. However, there will be many models of the description where there are other occurrences that are part of or external to the event. The PSL Ontology distinguishes between the occurrences that are conceptually part of an occurrence of a complex activity and the occurrences that are external to it as whether the occurrences are considered to be subactivity occurrences or not. The subactivity occurrence relation also links a complex activity occurrence object with the specific activity occurrences in the legal occurrence tree that are part of the complex activity occurrence. Figure 4 shows a subtree of an occurrence tree containing two occurrences of δ; o_1 and o_2. The subactivity occurrences on each occurrence are circled.

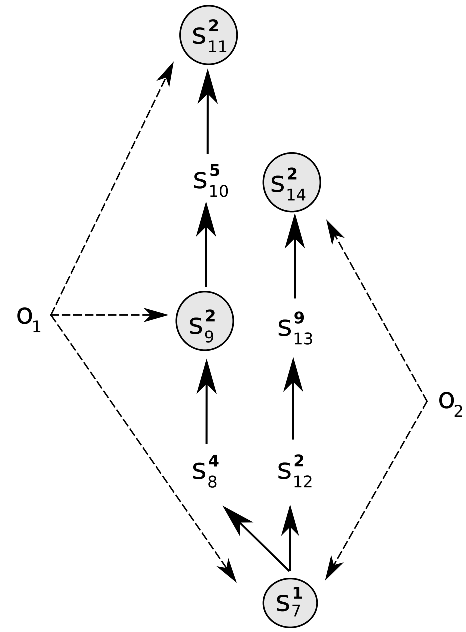

Figure 4: Two occurrences of δ in an occurrence tree. The circled occurrences represent the subactivity occurrences.

The dashed arrows between the occurrences of δ and the circled occurrences represent the relationship captured by the subactivity occurrence relation. The occurrences between the subactivity occurrences are external. Both branches contain two occurrences of the activity a_2. However, o_1 contains two subactivity occurrences of a_2 (s_9 and s_{11}), where as, o_2 has only one subactivity occurrence of a_2, s_{14}. The other occurrence of a_2 (s_{12}) on the branch on the right, is external to o_2. Therefore o_1 is not an intended occurrence of δ, but o_2 is. The closure constraints on the subactivity occurrences completes the definition of an activity in this sense. Say, a coffee making activity may involve an initial subactivity occurrence of coffee filter change as well as other subactivity occurrences, but we would like to conclude from a reasonable description of the activity that a second *subactivity occurrence* of filter change after the brewing started is not part of an occurrence of the activity.

The closure statements can be given as part of the activity descriptions. In the example above the activity description of the activity δ can be given as follows:

$$\forall o.occ\_of(o, \delta) \supset$$
$$\exists s_1, s_2.occ\_of(s_1, a_1) \wedge occ\_of(s_2, a_2) \wedge$$
$$subact\_occ(s_1, o) \wedge subact\_occ(s_2, o) \wedge \quad (5)$$
$$min\_prec(s_1, s_2, \delta) \wedge$$
$$(\forall s.subact\_occ(s, o) \supset s = s_1 \vee s = s_2)$$

This version of the process description eliminates all the unintended models. For example the occurrence in Figure 3.b where $s^2_{33}, s^1_{34}, s^1_{41}$ and s^2_{74} are subactivity occurrences is ruled out. Note that an occurrence with the root occurrence, s_{34}, and leaf occurrence, s_{74}, with all other occurrences in between being external is still consistent.

The closure constraints can be easy to specify if an activity's occurrences have a simple structure or have strong internal symmetries. In the example above, every occurrence of δ has the same number of subactivity occurrences; in fact they are isomorphic to each other and no activity has multiple occurrences. It is not hard to imagine how specifying the closure constraints get more complicated or tedious for activities whose occurrences are more distinct from each other and more sophisticated constraints like conditional occurrences, non-determinism and repetition must be used to describe the activity. Even after an activity is described correctly, any change in the behavior specification would require that the closure constraints to be changed accordingly. Therefore, it is convenient to impose a global closure constraint on activity occurrences, rather than having to close them individually.

This problem is analogous to the frame problem where a complete description of an activity requires the specification of the fluents that are *not* affected by the activity as well as the fluents that are affected by the activity. The activities' effects on the fluents are captured by effect axioms. Solutions to the frame problem provide mechanisms to infer the fluents that are unaffected by an occurrence of an activity from the effect axiom of the activity alone without the need for an explicit specification of the fluents that are unaffected by the activity.

Similarly, given a process description for an activity that specifies the subactivity occurrences that are part of the occurrences of the activity, we wish to conclude from this description alone that there is no other way that the activity can occur, i.e. rule out other occurrences that are not explicitly mentioned in the description. This similarity is not so surprising since a complex activity description can be seen as a "high level" effect axiom. The effect axiom for a primitive activity specifies the fluents that change as a result of its occurrence where a complex activity description specifies the activities that occur as a result of its occurrence which in turn affect the fluents as specified in their effect axioms.

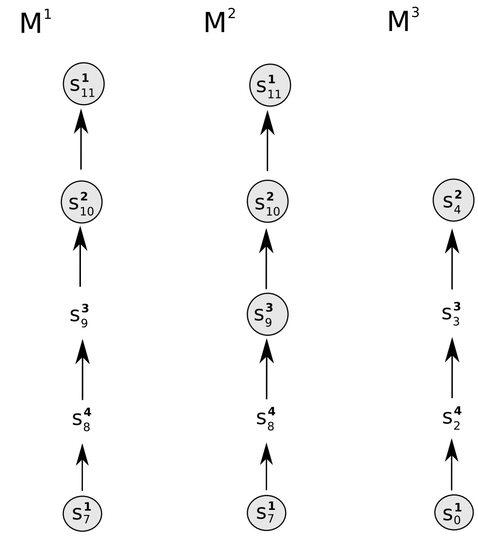

Figure 5:

The Minimization Policy

We propose a circumscription(McCarthy 1980)(McCarthy 1986) based approach to minimize the models of the process descriptions. First we introduce some definitions to characterize the minimization policy.

Definition 1 *Let $\mathcal{M}^1$ and $\mathcal{M}^2$ be models of $\mathcal{T}_{\rm psl} \cup \mathcal{T}_{\rm dt}$ where the set $\mathcal{T}_{\rm psl}$ denotes the PSL Ontology, $\mathcal{T}_{\rm dt}$ is a domain theory i.e. set of process descriptions and action theories for the primitive actions. A complex occurrence $o_1 \in \mathcal{O}^{\mathcal{M}^1}$ is strong occurrence-monomorphic to a complex occurrence $o_2 \in \mathcal{O}^{\mathcal{M}^2}$, where $\mathcal{O}$ denotes the set of activity occurrences in a model, iff:*

1. $\exists a \in \mathcal{T}_{\rm dt}. < o_1, a > \in {\rm occ\_of}^{\mathcal{M}^1} \wedge < o_2, a > \in {\rm occ\_of}^{\mathcal{M}^2}$
2. *There exists an injection* $\psi : \{s | < s, o_1 > \in {\rm subact\_occ}^{\mathcal{M}^1}\} \rightarrow \{s | < s, o_2 > \in {\rm subact\_occ}^{\mathcal{M}^2}\}$ *such that:*
$$s = \psi(s)$$

In the PSL Ontology the occurrence tree is fixed in every model of a domain. Therefore, the second condition in the definition requires that if $o_1^{\mathcal{M}^1}$ is strong occurrence-monomorphic to $o_2^{\mathcal{M}^2}$, then every subactivity occurrence of $o_1^{\mathcal{M}^1}$ is a subactivity occurrence of $o_2^{\mathcal{M}^2}$ as well. In other words o_1 can be embedded into o_2. Figure 5 shows three occurrences of the activity δ in different models.

The occurrence in $\mathcal{M}^1$ is strong occurrence-monomorphic to the occurrence in $\mathcal{M}^2$. Note that in these models δ occurs on the same branch of the occurrence tree because the occurrence objects on the branches are identical. The occurrence in $\mathcal{M}^3$ however is not strong occurrence-monomorphic to the occurrences in the other models, even though the activities that occur on the branch in $\mathcal{M}^3$ also exist in the other occurrences in the same order.

Definition 2 *A set of occurrences $\mathcal{S} \subset \{s|\,\text{subact\_occ}(s,o)\}$ where o is an occurrence of a complex activity, δ, is unnecessary with respect to an occurrence, o_1, iff o_1 is strong occurrence-monomorphic to o and $\mathcal{S} \cap \{s|\,\text{subact\_occ}(s,o_1)\} = \varnothing$.*

In other words, a set of subactivity occurrences of a complex activity occurrence is not necessary if and only if it is consistent that there is an occurrence of the activity that has the same subactivity occurrences in the same order except for the ones in the set.

Definition 3 *An occurrence, o_1, of a complex activity is preferred to another occurrence o_2 iff o_1 is strong occurrence-moomorphic to o_2 but o_2 is not strong occurrence-monomorphic to o_2.*

The following result trivially follows from the definitions above:

Corollary 1 *An occurrence o_1 is preferred to another occurrence o_2 iff there exists a set, $\mathcal{S}$, such that $\mathcal{S} \subset \{s|\,\text{subact\_occ}(s,o_2)\}$ and $\mathcal{S} \neq \varnothing$, and $\mathcal{S}$ is unnecessary with respect to o_1.*

Definition 4 *Let $\mathcal{M}^1$ and $\mathcal{M}^2$ be models of $\mathcal{T}_{\text{psl}} \cup \mathcal{T}_{\text{dt}}$, $\mathcal{M}^1$ is a preferred model to $\mathcal{M}^2$ iff $|\mathcal{O}^{\mathcal{M}^1}| = |\mathcal{O}^{\mathcal{M}^2}|$ and for every $o_1 \in \mathcal{O}^{\mathcal{M}^1}$ there exists a distinct $o_2 \in \mathcal{O}^{\mathcal{M}^2}$ such that o_1 is strong occurrence monomorphic to o_2 and there exists $o_3 \in \mathcal{O}^{\mathcal{M}^1}$ and $o_4 \in \mathcal{O}^{\mathcal{M}^2}$ such that o_3 is preferred to o_4.*

The minimization eliminates the models that contain subactivity occurrences that are not necessary to exist by the corresponding process descriptions. Intended models of a process description do not contain subactivity occurrences that are not necessary.

Definition 5 *A model of $\mathcal{T}_{\text{psl}} \cup \mathcal{T}_{\text{dt}}$, $\mathcal{M}^1$ is minimal if there is no model of $\mathcal{T}_{\text{psl}} \cup \mathcal{T}_{\text{dt}}$ that is preferred to $\mathcal{M}^1$.*

The circumscription policy that corresponds to the preference policy described above is as follows:

$$\text{Circ}(\mathcal{T}_{\text{psl}} \cup \mathcal{T}_{\text{dt}},; subact\_occ; min\_prec, root\_occ, leaf\_occ, next\_subocc) \quad (6)$$

The models where the extension of the *subact_occ* relation is minimal are preferred.

The extension of the *min_prec*, *root_occ*, *leaf_occ*, *next_subocc* relations are allowed to vary since they are relations on subactivity occurrences.

We provide below a model theoretic characterization of our circumscription policy following Lifschitz's results in (Lifschitz 1985) in order to be able to prove certain properties of the preferred models:

Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be two arbitrary models of $\mathcal{T}_{\text{psl}} \cup \mathcal{T}_{\text{pd}}$, the model $\mathcal{M}_1$ is preferred to the model $\mathcal{M}_2$ iff:

1. $\|\mathcal{M}_1\| = \|\mathcal{M}_2\|$
2. The interpretation of every function and predicate symbols that are not used to define the *subact_occ* relation or not defined in terms of *subact_occ* relation are the same.
3. $subact\_occ^{\mathcal{M}_1} \subset subact\_occ^{\mathcal{M}_2}$.

A model $\mathcal{M}$ is minimal if there is no other model that is preferred to it.

Note that the extension of the *occ_of* relation is fixed between the compared models. Given the primitive activities in a domain, the occurrences of the atomic activities are the same in every model of the PSL Ontology. They constitute the nodes of the occurrence tree. However, occurrences of the complex activities vary from model to model. It is consistent for a complex activity to not to occur at all in general. Since complex occurrences are composed of subactivity occurrences, letting complex occurrences vary while minimizing the subactivity occurrences would result in always preferring the models where no complex activity occur. Fixing the *occ_of* relations fixes the number of complex activity occurrences in the compared models, therefore ensures that a model is not preferred over another based on fewer complex activity occurrences.

A conditional activity occurrence may involve fewer subactivity occurrences in some situations than in others. The minimization policy should not conclude that such activities can only occur in situations where their subactivity occurrences are minimal. Models should be compared only if the occurrences appear in the same context. Moreover, if an occurrence of an activity in a model can be minimized into multiple non-isomorphic occurrences, there should be preferred models for all and only such minimal occurrences. In general, assuming that the intended models of a process description correspond to the intuition described above, for all the models of a process description that does not contain unnecessary subactivity occurrences there must be a corresponding circumscribed model and no circumscribed model of the process description should contain unnecessary subactivity occurrences. We prove next that the proposed minimization policy satisfies these properties.

Theorem 1 *Let $\mathcal{M}$ be a model of $\mathcal{T}_{\text{psl}} \cup \mathcal{T}_{\text{dt}}$ such that no complex activity occurrence in $\mathcal{M}$ contains unnecessary subactivity occurrences, then $\mathcal{M}$ is also a model of* (6).

Proof All we need to show is that there is no model $\mathcal{M}'$ that is preferred to $\mathcal{M}$ by the minimization policy. For the sake of contradiction lets assume that there is such $\mathcal{M}'$. Then $subact\_occ^{\mathcal{M}'} \subset subact\_occ^{\mathcal{M}}$.

Since the occurrence tree is fixed in all models and the complex activity occurrences are fixed in compared models, complex activity occurrences in $\mathcal{M}'$ must be embedded in the complex activity occurrences in $\mathcal{M}$. To see this let $< o^{\mathcal{M}}, \delta^{\mathcal{M}} > \in \text{occ\_of}^{\mathcal{M}}$ and $< s^{\mathcal{M}}, o^{\mathcal{M}} > \in \text{subact\_occ}^{\mathcal{M}}$ where $o, \delta,$ and s are constants in the language denoting a complex activity occurrence, a complex activity and a subactivity occurrence respectively. Then if $< s^{\mathcal{M}'}, o^{\mathcal{M}'} > \in$ $\text{subact\_occ}^{\mathcal{M}'}$ it must be the case that $s^{\mathcal{M}'} = s^{\mathcal{M}}$ and $o^{\mathcal{M}'} = o^{\mathcal{M}}$ since the interpretation of the occurrence of relation and activity occurrences are fixed between the models. Also since $subact\_occ^{\mathcal{M}'} \subset subact\_occ^{\mathcal{M}}$, if $< s^{\mathcal{M}}, o^{\mathcal{M}} > \notin \text{subact\_occ}^{\mathcal{M}}$ then $< s^{\mathcal{M}'}, o^{\mathcal{M}'} > \notin \text{subact\_occ}^{\mathcal{M}'}$.

By the assumption in the theorem that $\mathcal{M}$ contains no unnecessary subactivity occurrences and the fact that complex activity occurrences in $\mathcal{M}'$ must be embedded in the occurrences in $\mathcal{M}$, the only way the condition $subact\_occ^{\mathcal{M}'} \subset subact\_occ^{\mathcal{M}}$ can hold is some of the complex activity occurrences in $\mathcal{M}$ do not exists in $\mathcal{M}'$. As argued before, fixing the extension of the *occ_of* relation in the minimization policy avoids the number of complex activity occurrences to be a preference factor and every complex activity occurrence must have subactivity occurrences by the PSL Ontology. □

Theorem 2 *Let $\mathcal{M}$ be a model of (6), then $\mathcal{M}$ does not contain unnecessary subactivity occurrences.*

Proof It is easy to see that this property holds. Let $\mathcal{M}$ contains unnecessary subactivity occurrences and let $\mathcal{M}'$ be a model of $\mathcal{T}_{\text{psl}} \cup \mathcal{T}_{\text{dt}}$ such that $\mathcal{M}'$ is exactly like $\mathcal{M}$ except $\mathcal{M}'$ does not contain the unnecessary subactivity occurrences. By the definition of unnecessary subactivity occurrences, the model $\mathcal{M}'$ of $\mathcal{T}_{\text{psl}} \cup \mathcal{T}_{\text{dt}}$ clearly exists if $\mathcal{M}$ contains unnecessary subactivity occurrences. We reach the contradiction by observing that, if $\mathcal{M}'$ existed, then it would be the case that $subact\_occ^{\mathcal{M}'} \subset subact\_occ^{\mathcal{M}}$ and $\mathcal{M}'$ would be preferred to $\mathcal{M}$ by the minimization policy and $\mathcal{M}$ could not be a model of (6). □

Minimization of External Activity Occurrences

When reasoning with the complex activities, it is usually convenient to assume some restrictions on the occurrences of other activities that interleave with the occurrences of the complex activities. Minimizing the subactivity occurrences or using explicit closure statements for complex activities ensure that the occurrences that are part of a complex activity occurrence are only the intended ones by the occurrence description. However, this has no effect on the occurrences that are not subactivity occurrences of the activity. In this section, we will assume that the complex activity descriptions are *closed* either explicitly or with the circumscription policy discussed in the earlier section and focus on the issue with the external activity occurrences.

Consider the activity, δ, introduced in the previous section:

$$\begin{aligned}&\forall o.occ\_of(o,\delta) \supset \\ &\quad \exists s_1, s_2.occ\_of(s_1,a_1) \wedge occ\_of(s_2,a_2) \wedge \\ &\qquad subact\_occ(s_1,o) \wedge subact\_occ(s_2,o) \wedge \\ &\qquad min\_prec(s_1,s_2,\delta)\end{aligned} \tag{7}$$

Suppose the activities in the domain and their effect axioms are as follows:

$$activity(\delta) \wedge activity(a_1) \wedge activity(a_2) \wedge activity(a_3) \wedge activity(a_4) \tag{8}$$

$$subactivity(a_1,\delta) \wedge subactivity(a_2,\delta) \tag{9}$$

$$Poss(a_1, prior(s)) \supset holds(F_1, s) \tag{10}$$

$$Poss(a_2, prior(s)) \wedge holds(F_1, prior(s)) \supset holds(F_2, s) \tag{11}$$

$$Poss(a_3, prior(s)) \supset holds(\neg F_1, s) \tag{12}$$

$$Poss(a_4, prior(s)) \supset holds(\neg F_3, s) \tag{13}$$

Under the lack of knowledge about the other activities that may occur concurrently with δ, we would like to make the following general conclusions about the occurrences of δ:

$$\forall o, s.occ\_of(o,\delta) \wedge leaf\_occ(s,o) \supset holds(F_2,s) \tag{14}$$

$$\begin{aligned}&\forall o, s_1, s_2.occ\_of(o,\delta) \wedge root\_occ(s_1,o) \wedge \\ &\quad leaf\_occ(s_2,o) \wedge holds(F_3, prior(s_1)) \supset \\ &\qquad holds(F_3, s_2)\end{aligned} \tag{15}$$

However, none of these conclusions are entailed by the theory. In the models where a_3 occurs between a_1 and a_2, the fluent F_1 is false before the occurrence of the conditional activity a_2 and a_2 does not necessarily cause F_2 to hold. Likewise, in the models where a_4 occurs between a_1 and a_2 the fluent F_3 is false after the occurrence of δ.

Note that, the assumption that the activity is already closed only eliminates the models where a_3 and a_4 occur as subactivity occurrences of δ or the models where there are multiple occurrences of a_1 and a_2. Occurrences of a_3 and a_4, even other occurrences of a_1 and a_2 are immune to the closure constraint as long as they are not subactivity occurrences.

It is not hard to imagine useful applications of this kind of non-monotonic reasoning on complex activities. Such as web service discovery or enterprise modeling where the effects of the activities need to be identified in isolation before considering their role in more complex scenarios or when reasoning in environments where not every activity occurrence is observable.

The formal definition of external occurrences is given below:

Definition 6 *An atomic occurrence, s, is external to a complex occurrence, o, iff s is not a subactivity occurrence of o and s occurs between two subactivity occurrences of o.*

$$\forall s, o. external\_occ(s,o) \equiv \exists s_1, s_2, .next\_subocc(s_1,s_2,o) \wedge precedes(s_1,s) \wedge precedes(s,s_2) \quad (16)$$

The *next_subocc* relation holds if two subactivity occurrences of an occurrence are successive. The *precedes* relation holds between the occurrences that are members of the legal occurrence tree. Therefore only atomic activities can have external occurrences, complex activity occurrences are not members of the occurrence tree, however their atomic subactivity occurrences are. The definition simply considers every occurrence that is between two consecutive subactivity occurrences of an activity to be external to the activity.

It might be tempting to add an axiom to the theory that forces external activities to not exists at all.

$$\forall s, o. \neg external\_occ(s,o) \quad (17)$$

Unfortunately, this would be too strong. To see this lets modify the domain as follows:

$$activity(a_5) \quad (18)$$

$$Poss(a_5, prior(s)) \supset holds(F_4, s) \quad (19)$$

$$Poss(a_1, prior(s)) \supset holds(F_1, s) \wedge holds(\neg F_4, s) \quad (20)$$

$$Poss(a_2, s) \equiv holds(F_4, s) \quad (21)$$

The new activity a_5 has the effect of causing F_4 to be true and the activity a_2 now has the precondition that F_4 must be true. Also the activity a_1 has a new effect, it causes F_4 to be false.

It is easy to see that the occurrences of δ require an external occurrence of a_5 before the subactivity occurrence of a_2. Therefore eliminating the possibility of external activity occurrences causes δ to not occur. However, the occurrences of a_3 and a_4 are still unnecessary.

We would like to allow occurrences of external activities only when they are necessary for the complex activities to occur and conclude that no other external activities occur.

The Minimization Policy

Javier Pinto in his PhD Thesis (Pinto 1994) proposed a circumscription based preference policy for a partially ordered set of actions that are known to occur. The preference policy minimizes the occurrence of actions that are not known to occur. The correctness of the policy in the case of more expressive domain descriptions that included a form of conditional occurrences and ordering formulas with temporal constraints was argued using some of the common benchmark problems in artificial intelligence. The occurrence axioms in his framework is conceptually similar to the complex activity descriptions in PSL. The actions that are not explicitly stated to occur correspond to the external activities in our framework. However there are important differences. Unlike PSL the complex activities and their occurrences are not objects in the situation calculus framework that the work in (Pinto 1994) based on and the kinds of scenarios that that the minimization policy was argued to work correctly are much more limited that the classes of activities representable in PSL. Pinto's *occurs* predicate captures what *actually* occurs. Therefore in every model of the theory there is only one set of occurrences of the known actions which is conceptually occurrence of the complex action the domain describes. Also the actions occur as close to the initial situation as possible since every action that occur before the known actions are *actual* as well and treated as unintended.

In PSL, a model of a domain theory corresponds to consistent occurrences of possibly all the complex activities in the domain. Therefore in a model there may be multiple occurrences of multiple activities in various situations (some may be in the same situation). The minimization policy should preserve the occurrences in a model while eliminating the external occurrences, hence allow reasoning with possible occurrences of activities in models where interference from the external world is minimized.

A complex activity occurrence is preferred to another if the former occurs with the same root occurrence and has the same subactivities occur in the same order but has fewer unnecessary external activity occurrences. The notion of unnecessary external activities is similar to the notion of unnecessary subactivity occurrences described in the earlier section. The external activities in a preferred occurrence must be embedded in the external activities in the unpreferred occurrence. In other words if a complex activity occurrence in the same situation involves different external activity occurrences or the same external activity occurrences in a different order in different models, these models should not be comparable. Suppose the activity δ occurs in a situation where after a_1 occurs either the external activities a_6 and a_7 or the external activity a_9 must occur before a_2 can occur. Four of the possible models of the occurrences of δ is shown in Figure 6.

Our minimization policy considers the models $\mathcal{M}^1, \mathcal{M}^2$ and $\mathcal{M}^3$ to be incomparable. Although there is an extra external occurrence in $\mathcal{M}^3$ (the activity a_8) with respect to the model $\mathcal{M}^1$, the common external activities occur in a different order in these models. The model $\mathcal{M}^2$ has fewer external occurrences overall however the external activity it contains is distinct from the external activities in the other models. In these models the activity δ is considered to occur in different contexts. The only comparable models in the figure are the models $\mathcal{M}^3$ and $\mathcal{M}^4$ and clearly $\mathcal{M}^4$ is preferred to $\mathcal{M}^3$ since the model $\mathcal{M}^4$ shows that the occurrence of the external activity a_8 can simply be eliminated from the model $\mathcal{M}^3$ and the occurrence of δ would still be consistent.

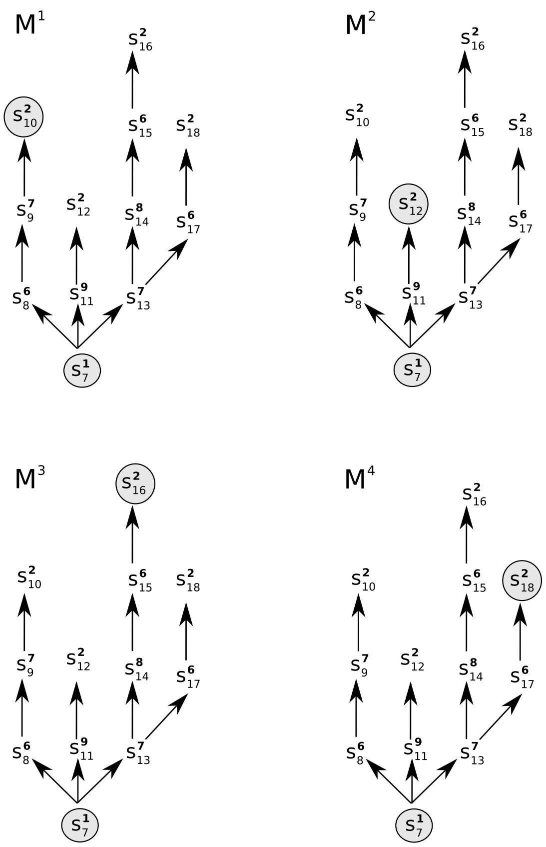

Figure 6:

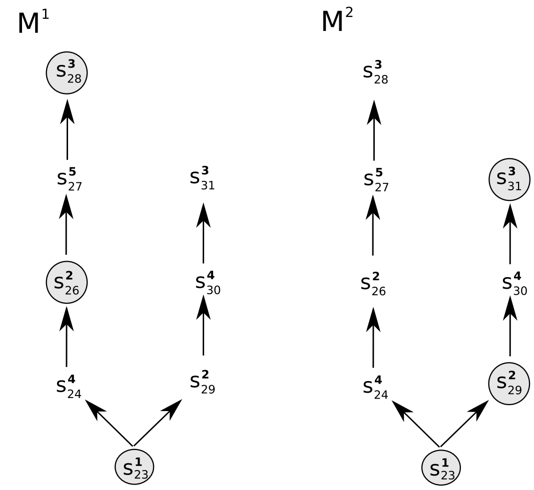

Figure 7:

In order for the complex occurrences to be considered to appear in the same context in different models, the external activities they contain must also occur between the same subactivity occurrences. Suppose the activity δ also requires a subactivity occurrence of a_3 after a_1. Figure 7 shows two possible models of this new version of the activity δ.

Suppose the two branches in the models are the only possible way the activity can occur with the root occurrence s_{23}. Then both of these models are preferred and they are not comparable. The models suggest that if the external activity a_4 occurs before a_2 then another external activity a_5 must occur before a_3 can occur however if a_4 occurs after a_2 then a_5 need not occur. It is easy to see that we cannot arrive at $\mathcal{M}^2$ from $\mathcal{M}^1$ by simply eliminating an external occurrence. When an external activity can occur with respect to the subactivity occurrences should not be determined by the minimization policy. The model $\mathcal{M}^1$ represents a distinct way the world can unfold in the context of an occurrence of δ and the occurrence of a_5 is necessary for that scenario to occur.

Finally, we would like to emphasize that in the compared models, the corresponding complex activity occurrences must be isomorphic and they must have the same root occurrence. Consider the models of the activity δ in Figure 8.

The occurrences of δ in $\mathcal{M}_1$ and $\mathcal{M}_2$ are not isomorphic and in the model $\mathcal{M}_3$ the occurrence starts in a different situation than in the other models. Therefore all three models are incomparable.

To provide a formal definition of the minimization policy first we define some useful concepts.

Definition 7 *Let $\mathcal{M}^1$ and $\mathcal{M}^2$ be models of $\mathcal{T}_{\rm psl} \cup \mathcal{T}_{\rm dt}$. A complex occurrence $o_1 \in \mathcal{O}^{\mathcal{M}^1}$ is contextual-*

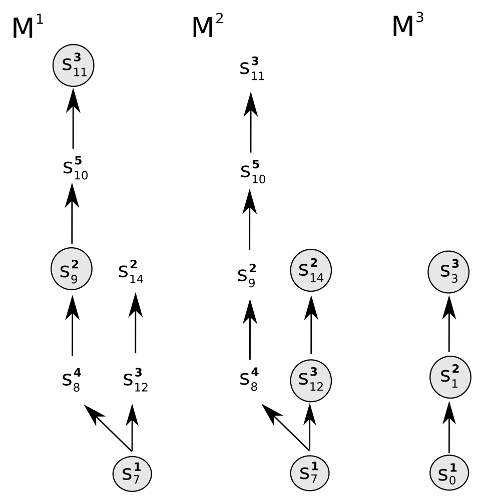

Figure 8:

occurrence isomorphic to a complex occurrence $o_2 \in \mathcal{O}^{\mathcal{M}^2}$ *iff:*

1. $\exists a \in \mathcal{T}_{\mathrm{dt}}. < o_1, a > \in \mathrm{occ\_of}^{\mathcal{M}^1} \wedge < o_2, a > \in \mathrm{occ\_of}^{\mathcal{M}^2}$
2. $\exists s \in \mathcal{O}^{\mathcal{M}^1}. < s, o_1 > \in \mathrm{root\_occ}^{\mathcal{M}^1} \wedge < s, o_2 > \in \mathrm{root\_occ}^{\mathcal{M}^2}$
3. *There exists an bijection* $\omega : \{s| < s, o_1 > \in \mathrm{subact\_occ}^{\mathcal{M}^1}\} \rightarrow \{s| < s, o_2 > \in \mathrm{subact\_occ}^{\mathcal{M}^2}\}$ *such that:*
   1. $\forall a \in \mathcal{T}_{\mathrm{dt}}. < s, a > \in \mathrm{occ\_of}^{\mathcal{M}^1} \equiv < \omega(s), a > \in \mathrm{occ\_of}^{\mathcal{M}^2}$
   2. $\exists a \in \mathcal{T}_{\mathrm{dt}}. < o_1, a > \in \mathrm{occ\_of}^{\mathcal{M}^1} \wedge < s_1, s_2, a > \in \mathrm{min\_prec}^{\mathcal{M}^1} \equiv < \omega(s_1), \omega(s_2), a > \in \mathrm{min\_prec}^{\mathcal{M}^2}$

In other words, two occurrences in different models are contextual-occurrence isomorphic iff they are the occurrences of the same complex activity, they have the *same* root occurrence object, and they have the same subactivities occurring in the same order.

Definition 8 *A complex occurrence* o_1 *in a model* $\mathcal{M}^1$ *is external-monomorphic to a complex occurrence* o_2 *in a model* $\mathcal{M}^2$ *iff*

1. $o_1\,\mathrm{is\,contextual-isomorphic\,to}\,o_2$.
2. *There exists an injection* $\varphi : \{s| < s, o_1 > \in \mathrm{external\_occ}^{\mathcal{M}^1}\} \rightarrow \{s| < s, o_2 > \in \mathrm{external\_occ}^{\mathcal{M}^2}\}$ *such that:*
   1. $\exists a \in \mathcal{T}_{\mathrm{dt}}. < s, a > \in \mathrm{occ\_of}^{\mathcal{M}^1} \wedge < \varphi(s), a > \in \mathrm{occ\_of}^{\mathcal{M}^2}$
   2. $< s_1, s_2 > \in \mathrm{precedes}^{\mathcal{M}^1} \equiv < \varphi(s_1), \varphi(s_2) > \in \mathrm{precedes}^{\mathcal{M}^2}$
   3. $\forall s_1, s_2, s \in \mathcal{O}^{\mathcal{M}^1}. < s_1, s_2, o_1 > \in \mathrm{next\_subocc}^{\mathcal{M}^1} \wedge$
   $< s_1, s >, < s, s_2 > \mathrm{precedes}^{\mathcal{M}^1} \equiv$
   $< \omega(s_1), \varphi(s) >, < \varphi(s), \omega(s_2) > \in \mathrm{precedes}^{\mathcal{M}^2}$

The models $\mathcal{M}^3$ and $\mathcal{M}^4$ in Figure 6 give an example of external-monomorphic occurrences. The occurrences are contextual-isomorphic and the external occurrences of the occurrence in $\mathcal{M}^4$ can be mapped into the external occurrences in the occurrence in $\mathcal{M}^3$. The mapping not only preserves the ordering of the external activities but also the subactivity occurrences that they occur between.

Definition 9 *A set of occurrences,* $\mathcal{S} \subset \{s|\,\mathrm{external\_occ}(s, o)\}$ *where* o *is a an occurrence of a complex activity, is unnecessary iff there exists an occurrence of the same activity,* o_1*, such that* o_1 *is external-monomorphic to* o *and* $\mathcal{S} \cap \{s|\,\mathrm{external\_occ}(s, o_1)\} = \varnothing$.

The next definition formalizes our notion of the preferred models.

Definition 10 *Let* $\mathcal{M}^1$ *and* $\mathcal{M}^2$ *be models of* $\mathcal{T}_{\mathrm{psl}} \cup \mathcal{T}_{\mathrm{dt}}$*,* $\mathcal{M}^1$ *is a preferred model to* $\mathcal{M}^2$ *iff for every occurrence,* o_2*, in* $\mathcal{M}^2$ *there exists an occurrence,* o_1*, in* $\mathcal{M}^1$ *such that* o_1 *is external-monomorphic to* o_2 *and every occurrence in* $\mathcal{M}^1$ *is external-monomorphic to some occurrence in* $\mathcal{M}^2$ *and there is an occurrence in* $\mathcal{M}^2$ *that is not external-monomorphic to an occurrence in* $\mathcal{M}^1$.

Definition 11 *A model of* $\mathcal{T}_{\mathrm{psl}} \cup \mathcal{T}_{\mathrm{dt}}$*,* $\mathcal{M}^1$ *is minimal iff there is no model of* $\mathcal{T}_{\mathrm{psl}} \cup \mathcal{T}_{\mathrm{dt}}$ *that is preferred to* $\mathcal{M}^1$*. Alternatively,* $\mathcal{M}^1$ *is minimal iff no occurrence in* $\mathcal{M}^1$ *contains unnecessary occurrences.*

Simply circumscribing the external activities would have the same effect as axiomatically ruling out the activities that occur externally. The minimization policy should preserve the isomorphic occurrences. Note that fixing subactivity occurrences would not achieve the intended result even if the complex occurrences and their roots are also fixed. Consider the models $\mathcal{M}^3$ and $\mathcal{M}^4$ in Figure 6 (repeated here in Figure 9).

The minimization policy should prefer the model $\mathcal{M}^4$ over the model $\mathcal{M}^3$. The extension of the relevant relations in $\mathcal{M}^3$ are:

$$< o_1, \delta > \in occ\_of^{\mathcal{M}^3} \tag{22}$$

$$< s_7, o_1 >, < s_{16}, o_1 > \in subact\_occ^{\mathcal{M}^3} \tag{23}$$

$$< s_7, o_1 > \in root\_occ^{\mathcal{M}^3} \tag{24}$$

It is consistent that the occurrence o_1 is also an occurrence of δ with the root s_7 in the model $\mathcal{M}^4$. However since the occurrence tree is fixed for all models, two atomic occurrences are the same object in different models if they have the exact same history. The second

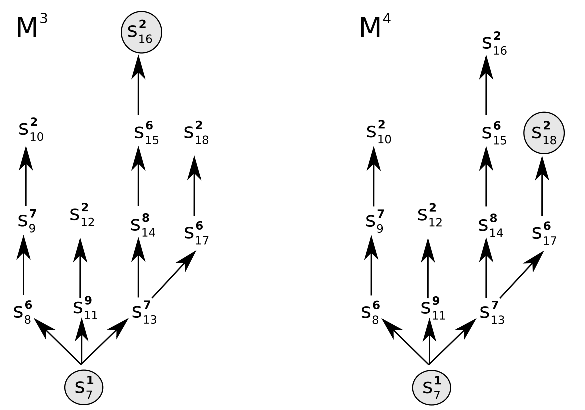

Figure 9:

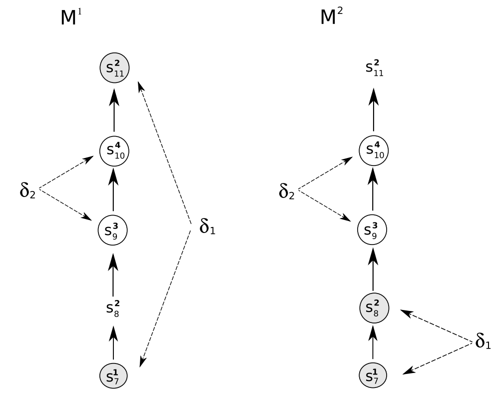

Figure 10:

subactivity occurrence in $\mathcal{M}^4$ appears in a different part of the occurrence tree and can never be unified with the second subactivity occurrence in the model $\mathcal{M}^3$. Likewise, the external activity occurrences in $\mathcal{M}^4$ cannot be embedded in the external activity occurrences in $\mathcal{M}^3$.

Javier Pinto in (Pinto 1994) introduces a second occurrence predicate, $occurs_{\mathcal{T}}$, that uses time points instead of situations. Formally,

$$occurs_{\mathcal{T}}(a,t) \equiv (\exists s).occurs(a,s) \wedge start(do(a,s)) = t \tag{25}$$

where *start*(do(*a*,*s*)) represents the time at which the activity ends. The preference policy minimizes the $occurs_{\mathcal{T}}$ relation and lets the *occurs* relation vary. Therefore, if a model, $\mathcal{M}^1$ exists with fewer occurrences compared to another model, $\mathcal{M}^2$, and if the overlapping activities occur in the same order in the compared models it is assumed that there exists a model, $\mathcal{M}^{1'}$, isomorphic to $\mathcal{M}^1$ with respect to the occurrences where the time points of the corresponding occurred activities in the models $\mathcal{M}^2$ and $\mathcal{M}^{1'}$ are unified, i.e. $\text{occurs}_{\mathcal{T}}^{\mathcal{M}^{1'}} \subset \text{occurs}_{\mathcal{T}}^{\mathcal{M}^2}$. However using time points when comparing models imposes some restrictions on the representation of activity durations. For example if the domain theory imposes a constraint that every activity takes 10 seconds to complete, the models $\mathcal{M}^3$ and $\mathcal{M}^4$ in figure 4 cannot be compared because different number of external activities occur between the subactivity occurrences of a_1 and a_2 in these models. In other words the ending time of the occurrence of a_2 in $\mathcal{M}^3$ must be greater then the ending time of the corresponding occurrence of a_2 in $\mathcal{M}^4$.

In our framework there are possibly multiple occurrences of multiple activities in the models that are minimized which imposes even more severe restrictions on the representation if time points are used in the minimization. Consider the models of the occurrences of two activities δ_1 and δ_2 depicted in Figure 10.

The occurrence of δ_2 and the first occurrence of a_2 are external with respect to the occurrence of δ_1 in the model $\mathcal{M}_1$. The model $\mathcal{M}_2$ should be preferred to the model $\mathcal{M}_1$ according to our preference policy. However fixing the time points of subactivity occurrences would impose a constraint on the occurrence order of the activities δ_1 and δ_2 since they appear on the same branch (intuitively the same time line). Therefore the models $\mathcal{M}_1$ and $\mathcal{M}_2$ would not be comparable. An intermediate model may exist where there is another occurrence of a_2 rooted at s_7 but on a different branch (after s_8 perhaps) where the external activities can be embedded. Such a model, if existed, would eliminate $\mathcal{M}_1$ from preferred models of the theory, in fact would in turn be discarded itself in favor of $\mathcal{M}_2$. However, in general such an intermediate model may not exist.

We introduce two new relations in order to define a circumscription policy that corresponds to the preference policy described above that works correctly in the presence of multiple occurrences of multiple activities in the domain without imposing restrictions on the durations of the activities.

The first relation *subact_order(a,o,n)* associates each subactivity, a, that occur as part of a complex occurrence, o, with a timepoint object, n, such that the ordering of the subactivities with respect to their associated timepoint corresponds to the sequence of subactivities in the occurrence. The axiomatization of the relation is given in Figure 11. Since the subactivities that occur as part of an occurrence are not necessarily distinct(same subactivity can occur many times), an auxiliary relation, *subact_occ_order(s,o,n)*, is defined first that maps each subactivity occurrence (which are distinct), s, of a complex occurrence, o, to an timepoint, n, that preserves the subactivity ordering. The timepoint objects in PSL-Core form an infinite discrete linear ordering. The ordering relation associated with the timepoint objects is the *before(t_1, t_2)* relation of PSL. The ordering of timepoints in each model is fixed and corresponds to the extension of the before relation in the model. The timepoints are intended to represent a time line. The *beginof* and *endof* functions on occurrences associate each occurrence with a beginning

Subact_occ_order is a relation among complex occurrences, their subactivity occurrences and timepoints.

$$\forall s, o, n.subact\_occ\_order(s, o, n) \supset occ\_of(s, o) \wedge timepoint(n) \quad (26)$$

The *subact_occ_order* relation associates every subactivity occurrence on branches of activity trees with a timepoint such that the associated timepoints must respect the ordering of the subactivity occurrences within the occurrence.

$$\forall s_1, s_2, o.min\_prec(s_1, s_2, o) \supset \exists n_1, n_2.subact\_occ\_order(s_1, o, n_1) \wedge subact\_occ\_order(s_2, o, n_2) \wedge before(n_1, n_2) \quad (27)$$

Every subactivity occurrence is associated with a unique number within the complex occurrence that it belongs to.

$$\forall s, o, n_1, n_2.subact\_occ\_order(s, o, n_1) \wedge subact\_occ\_order(s, o, n_2) \supset n_1 = n_2 \quad (28)$$

The *Subact_order* is a relation among complex occurrences, subactivities that occur as part of them and timepoints such that the subactivities are associated with the same timepoint values as their occurrences.

$$\forall a, o, n.subact\_order(a, o, n) \equiv \exists s.subact\_occ(s, o) \wedge occ\_of(s, a) \wedge subact\_occ\_order(s, o, n) \quad (29)$$

Figure 11: The axiomatization of the *subact_order* relation.

and ending timepoint respectively. Timepoints associated with occurrences on the same branch are naturally subject to ordering constraints. However, the timepoint objects that are parameters to the relations defined in Figures 11 and 12, are independent from the timepoints that represent the underlying time line, and are not subject to global temporal constraints. Intuitively, the new relations provide every complex occurrence with its own time line independent from its context on the occurrence tree or other complex occurrences that may interleave with them.

The second relation *extact_order(a,o,n)* associates each external activity, a, with respect to a complex occurrence, o, with a timepoint, n, such that the ordering of the external activity occurrences within the complex occurrence corresponds to the ordering of the timepoints associated with them. The axiomatization of the relation is given in Figure 12. Similar to the

Extact_occ_order is a relation among complex activity occurrences, their external occurrences and timepoints.

$$\forall s, o, n.extact\_occ\_order(s, o, n) \supset external\_occ(s, o) \wedge timepoint(n) \quad (30)$$

Occurrences external with respect to a complex occurrence are associated with a timepoint preserves their relative ordering.

$$\forall s_1, s_2, o.external\_occ(s_1, o) \wedge external\_occ(s_2, o) \wedge prec(s_1, s_2, o) \supset \exists n_1, n_2.extact\_occ\_order(s_1, o, n_1) \wedge extact\_occ\_order(s_2, o, n_2) \wedge before(n_1, n_2) \quad (31)$$

The timepoint associated with an external occurrence is unique within the complex occurrence it appears.

$$\forall s, o, n_1, n_2.extact\_occ\_order(s_1, o, n_1) \wedge extact\_occ\_order(s_2, o, n_2) \supset n_1 = n_2 \quad (32)$$

The timepoint associated with an external activity must be between the timepoints associated with the subactivity occurrences that come before and after it.

$$\forall s_1, s_2, s_3, o, n_1, n_2, n_3.subact\_occ\_order(s_1, o, n_1) \wedge subact\_occ\_order(s_2, o, n_2) \wedge extact\_occ\_order(s_3, o, n_3) \wedge prec(s_1, s_3) \wedge prec(s_3, s_2) \supset before(n_1, n_3) \wedge before(n_3, n_2) \quad (33)$$

The timepoints of the subactivity occurrences and external occurrences of a complex occurrence are distinct.

$$\forall s_1, s_2, o, n_1, n_2.subact\_occ\_order(s_1, o, n_1) \wedge extact\_occ\_order(s_2, o, n_2) \supset n_1 \neq n_2 \quad (34)$$

Extact_order is a relation among complex occurrences, the activities that occur external to them and timepoints such that the external activities are associated with the same timepoints as their occurrences.

$$\forall a, o, n.extact\_order(a, o, n) \equiv \exists s.occ\_of(s, a) \wedge subact\_occ\_order(s, o, n) \quad (35)$$

Figure 12: The axiomatization of the *extact_order* relation.

definition of *subact_order(a,o,n)* relation, an auxiliary relation, *Extact_occ_order* , is defined first on the occurrences of external activities. The axioms ensure that the assigned timepoints to the occurrences within a complex activity occurrence preserve the way external occurrences interleave with the subactivity occurrences.

First we will present the circumscription policy that "almost" corresponds the the preference policy we propose, then we will show how to obtain the minimal occurrences from the models of the circumscription policy.

$$\mathrm{Circ}(\mathcal{T}_{\mathrm{psl}} \cup \mathcal{T}_{\mathrm{dt}}, ; extact\_order; min\_prec, next\_subocc, subact\_occ, leaf\_occ) \quad (36)$$

The domain theory, $\mathcal{T}_{\mathrm{dt}}$, may or may not be already minimized with respect to the activity trees. In the compared models, the activity trees are equivalent up to isomorphism. Therefore the minimization of external activities is independent from the minimization of activity trees. The extensions of the *min_prec*, *next_subocc*, *subact_occ* and *leaf_occ* relations are allowed to vary. This gives the freedom to the circumscription policy to compare the isomorphic models with respect to the activity trees. The extension of *subact_order*, *root_occ* and *occ_of* relations are fixed. Fixing the *subact_order* relation preserves the subactivity occurrence ordering in the compared models, the *root_occ* and *occ_of* relations ensure the contextual equivalence in the compared models.

Following (Lifschitz 1985), a model theoretic characterization of the circumscription policy can be given as follows:

Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be two arbitrary models of $\mathcal{T}_{\mathrm{psl}} \cup \mathcal{T}_{\mathrm{pd}}$, the model $\mathcal{M}_1$ is preferred to the model $\mathcal{M}_2$ iff:

1. $\| \mathcal{M}_1 \| = \| \mathcal{M}_2 \|$

2. The interpretation of every function and predicate symbols except for min_prec,next_subocc, subact_occ,leaf_occ are the same.

3. $extact\_order^{\mathcal{M}_1} \subset extact\_order^{\mathcal{M}_2}$

A model $\mathcal{M}$ is minimal if there is no other model that is preferred to it.

We can now show that the model $\mathcal{M}^4$ in Figure 9 (repeated here again in Figure 13) is preferred over the model $\mathcal{M}^3$ by the circumscription policy.

It is easy to see that we can obtain the following

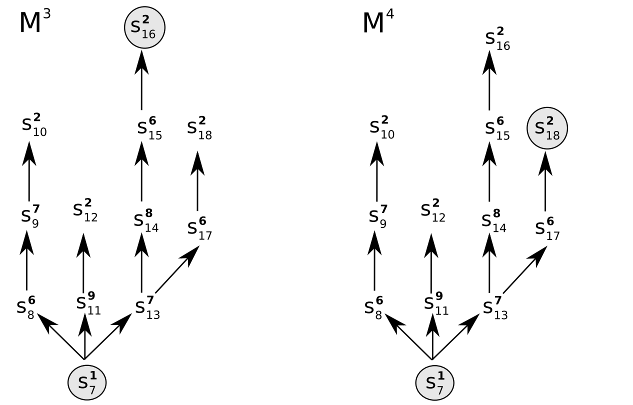

Figure 13:

results from the models:

$$\mathcal{M}^3 \models \exists o, t_1, t_2, t_3, t_4, t_5 . occ\_of(o, \delta) \wedge root\_occ(s_7, o) \wedge subact\_order(a_1, o, t_1) \wedge subact\_order(a_2, o, t_2) \wedge extact\_order(a_7, o, t_3) \wedge extact\_order(a_8, o, t_4) \wedge extact\_order(a_6, o, t_5) \wedge t_1 < t_3 < t_4 < t_5 < t_2 \quad (37)$$

$$\mathcal{M}^4 \models \exists o, t_1, t_2, t_3, t_4 . occ\_of(o, \delta) \wedge root\_occ(s_7, o) \wedge subact\_order(a_1, o, t_1) \wedge subact\_order(a_2, o, t_2) \wedge extact\_order(a_7, o, t_3) \wedge extact\_order(a_6, o, t_4) \wedge t_1 < t_3 < t_4 < t_2 \quad (38)$$

Based on these results we can prove that $\mathcal{M}^4$ is preferred to $\mathcal{M}^3$ by the circumscription policy given that the formulas (37) and (38) capture the only ways these two models differ, i.e. they agree on everything else (ceteris paribus).

Theorem 3 *Ceteris paribus, a model with the properties as in the formula* (38) *is preferred to a model with the properties as in the formula* (37) *according to the circumscription policy in* (45).

Proof We show that all three conditions in the model theoretic characterization of the circumscription policy is consistent to be satisfied by the models.

$\| \mathcal{M}^3 \| = \| \mathcal{M}^4 \|$ This is inferred by the ceteris paribus assumption.

Fixed functions and predicates The *subact_order*, *root_occ* and *occ_of* relations are not entirely covered by the ceteris paribus assumption since they appear in (37) and (38). Therefore it needs to be shown that their extensions are consistent to be the same.

Both models have an occurrence of δ, it is consistent that these occurrences are the same objects. The root occurrence of this occurrence is explicitly fixed in (37) and (38) (s_7). Let o be the occurrence of δ whose existence is suggested by both of the models.

i.e. we are assuming that they are both referring to this very 'o', since it is consistent to assume so. Then:

$$< a_1, o, t_1^{\mathcal{M}^3} > \in \text{subact\_order}^{\mathcal{M}^3}$$

$$< a_2, o, t_2^{\mathcal{M}^3} > \in \text{subact\_order}^{\mathcal{M}^3}$$

$$< t_1^{\mathcal{M}^3}, t_2^{\mathcal{M}^3} > \in \text{before}^{\mathcal{M}^3}$$

$$< a_1, o, t_1^{\mathcal{M}^4} > \in \text{subact\_order}^{\mathcal{M}^4}$$

$$< a_2, o, t_2^{\mathcal{M}^4} > \in \text{subact\_order}^{\mathcal{M}^4}$$

$$< t_1^{\mathcal{M}^4}, t_2^{\mathcal{M}^4} > \in \text{before}^{\mathcal{M}^4}$$

where $t_i^{\mathcal{M}^j}$ is an object that $\mathcal{M}^j$ considers to be named t_i in the theory. In other words they agree on everything except 'possibly' for what they mean by t_1 and t_2 (including their order). Since every occurrence gets to have their own timepoints independent from other occurrences and the underlying timeline there is no external constraint imposed on the models as to how to interpret the timepoints, so there is nothing that rules out that the models are referring to the same timepoint objects.

The extension of extact_order The extension of the extact_order and before relations in the models is as follows:

$$< a_7, o, t_3^{\mathcal{M}^3} > \in \text{extact\_order}^{\mathcal{M}^3}$$

$$< a_8, o, t_4^{\mathcal{M}^3} > \in \text{extact\_order}^{\mathcal{M}^3}$$

$$< a_6, o, t_5^{\mathcal{M}^3} > \in \text{extact\_order}^{\mathcal{M}^3}$$

$$< t_3^{\mathcal{M}^3}, t_4^{\mathcal{M}^3} > \in \text{before}^{\mathcal{M}^3}$$

$$< t_4^{\mathcal{M}^3}, t_5^{\mathcal{M}^3} > \in \text{before}^{\mathcal{M}^3}$$

$$< a_7, o, t_3^{\mathcal{M}^4} > \in \text{extact\_order}^{\mathcal{M}^4}$$

$$< a_6, o, t_4^{\mathcal{M}^4} > \in \text{extact\_order}^{\mathcal{M}^4}$$

$$< t_3^{\mathcal{M}^4}, t_4^{\mathcal{M}^4} > \in \text{before}^{\mathcal{M}^4}$$

As discussed above it is consistent that the matching timepoint constants are interpreted the same way by the models. Then it is consistent that,

$$\text{extact\_order}^{\mathcal{M}^4} \subset \text{extact\_order}^{\mathcal{M}^3}$$

□

Saying that the model $\mathcal{M}^4$in Figure (13) is preferred to $\mathcal{M}^3$ by the circumscription policy, though convenient, is not very precise. Because, other complex occurrences, possible elsewhere in the occurrence tree, and the timepoint objects associated with the occurrences are not shown in the figure. What we mean by such a statement is that, if the (partial) models are exactly the same else where in their occurrence trees (ceteris paribus), for every model $\mathcal{M}^{3'}$ that completes the interpretation of $\mathcal{M}^3$ with specific timepoints there exists a model $\mathcal{M}^{4'}$ that completes $\mathcal{M}^4$ with the corresponding timepoints interpreted the same way. This result follows from the theorem (3). Then no such $\mathcal{M}^{3'}$ can be a model of the circumscription policy.

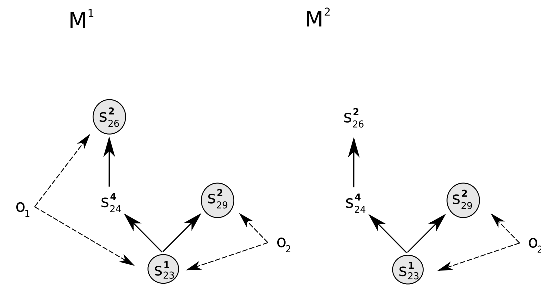

Figure 14:

Fixing the complex occurrences in the minimization policy has an undesired effect when there are more isomorphic occurrences of an activity on the same root in a model then there can exist minimally. To see this consider the models of the activity δ in Figure (14).

The occurrence s_{24} in the model $\mathcal{M}^1$ is external with respect to the complex occurrence o_1. The other occurrence, o_2, in the model shows that the subactivity occurrence of a_2 can immediately follow the root occurrence of a_1. Therefore the occurrence o_1 is not minimal. Note that in an occurrence tree, successive occurrences of an occurrence must be occurrences of distinct activities. Then no model of the activity with two occurrences rooted at s_{23} can avoid a non-minimal occurrence. The model $\mathcal{M}^2$ does not contain a non-minimal occurrence, however because the minimization fixes the complex occurrences and root occurrences, $\mathcal{M}^2$ is not comparable to $\mathcal{M}^1$.

The circumscription policy minimizes the activity trees. An activity tree in a model of the circumscribed theory is minimal in the sense that no other model of the activity tree with the same root occurrence and same number of branches can be embedded into it. If an occurrence in a minimized activity tree is not minimal then there exists a minimal occurrence that is preferred to it in the same activity tree. Note that if this is not the case the model that contains the activity tree would be preferred to another model where the activity tree has the property.

Based on this observation we next define a relation on complex occurrences, *minimal*, such that an occurrence is minimal if there is no occurrence in the same activity tree that is preferred to it. First we introduce some auxiliary relations to define the *minimal* relation.

Definition 12 *The occurrence o_1 of an activity δ is branch-monomorphic to an occurrence o_2 iff they are branches of the same activity tree and the occurrences in o_1 can be embedded into o_2.*

$$branch-mono(o_1,o_2,\delta) \equiv (\exists s.root\_occ(s,o_1) \wedge root\_occ(s,o_2)) \wedge (\forall s_1.subact\_occ(s_1,o_1) \supset \exists s_2.subact\_occ(s_2,o_2) \wedge mono(s_1,s_2,\delta)) \wedge (\forall s_1,s_2,s_3,s_4.subact\_occ(s_1,o_1) \wedge subact\_occ(s_2,o_1) \wedge subact\_occ(s_3,o_2) \wedge subact\_occ(s_4,o_2) \wedge mono(s_1,s_3,\delta) \wedge mono(s_2,s_4,\delta) \wedge min\_prec(s_1,s_2,\delta) \supset min\_prec(s_3,s_4,\delta)) \quad (39)$$

The *mono* relation in the PSL Ontology defines a one to one mapping of the subactivity occurrences between the two occurrences of a complex activity. The mono relation does not necessarily define a total mapping or preserve the ordering of the subactivities that occur on the branches however it is a unique between any two occurrences of an activity. The *branch-mono* relation is total with respect to the first occurrence and the mapped occurrences are on the same activity tree. The relation requires that the subactivity occurrence ordering is also preserved between the branches.

Definition 13 *Two occurrences of an activity on the same activity tree are branch-isomorphic iff there is a one to one and onto mapping between the branches that preserves the subactivity occurrence ordering.*

$$branch-iso(o_1,o_2,\delta) \equiv branch-mono(o_1,o_2,\delta) \wedge branch-mono(o_2,o_1,\delta) \quad (40)$$

Definition 14 *A sister activity, δ', of an activity, δ, has an occurrence, o', that correspond to every occurrence, o, of δ such that every subactivity occurrence as well as external occurrence of o is a subactivity occurrence of o'.*

$$sister(\delta',\delta) \equiv \forall o.occ\_of(o,\delta) \supset \exists o'.occ\_of(o',\delta') \wedge (\forall s.subact\_occ(s,o) \vee external\_occ(s,o) \equiv subact\_occ(s,\delta')) \quad (41)$$

Definition 15 *Sister occurrences are the corresponding occurrences of an activity and its sister.*

$$sister\_occ(o',o) \equiv \exists \delta',\delta.sister(\delta',\delta) \wedge occ\_of(o',\delta') \wedge occ\_of(o,\delta) \wedge (\exists s_1,s_2.root\_occ(s_1,o) \wedge root\_occ(s_1,o') \wedge leaf\_occ(s_2,o) \wedge leaf\_occ(s_2,o')) \quad (42)$$

Now we can give a first-order definition for a version of the external-monomorphism defined earlier (definition (8)). The earlier version is a function between occurrences in different models of the theory. The version we define here holds between the occurrences of an activity tree in the same model.

Definition 16 *An occurrence, o_1, is external-mono to an occurrence, o_2, iff the occurrences are branch–isomorphic and the external occurrences in o_1 can be embedded in to the external occurrences in o_2 preserving their ordering in the occurrences.*

$$external-mono(o_1,o_2) \equiv branch-iso(o_1,o_2) \wedge \exists o_1',o_2',\delta'.sister\_occ(o_1',o_1) \wedge sister\_occ(o_2',o_2) \wedge branch-mono(o_1',o_2',\delta') \wedge \forall s_1,s_2,s.next\_subact(s_1,s_2,o_1) \wedge precedes(s_1,s) \wedge precedes(s,s_2) \supset \exists s_3,s_4,s'.next\_subact(s_3,s_4,o_2) \wedge mono(s_1,s_3,\delta) \wedge mono(s_2,s_4,\delta) \wedge mono(s,s',\delta') \wedge precedes(s_3,s') \wedge precedes(s',s_4) \quad (43)$$

We can obtain the minimal occurrences in an activity tree using the defined relations above.

Definition 17 *An occurrence in an activity tree is minimal iff no other occurrence exists in the tree that is preferred to it.*

$$minimal(o) \equiv \neg\exists o_1.external-mono(o_1,o) \quad (44)$$

The relation *minimal*, is a defined relation. Therefore it does not eliminate models of the theory. It just selects the *most minimal* occurrences out of all the isomorphic occurrences in an activity tree in a model with respect to the preference policy defined in this section. In other words if *minimal(o)* holds for some o, then there is no o' in the same activity tree that is preferred to it. In Figure (14), *external_mono(o_2, o_1)* holds in $\mathcal{M}^1$, however *external_mono(o_1, o_2)* does not hold. Therefore *minimal(o_2)* holds. Note that in $\mathcal{M}^2$, *minimal(o_2)* is trivially true.

In general the most minimal occurrence out of all the comparable occurrences in an activity tree is not necessarily minimal with respect to other possible models. However models of the circumscribed theory ensures that minimal occurrences in activity trees are also minimal overall occurrences. Therefore the *minimal* predicate selects only the intended occurrences of complex activities in the minimal models of a theory.

Given a domain theory possibly containing open form process descriptions as described in section () both minimization policies; minimization of activity trees and minimization of external activities can be applied together by combining the corresponding circumscription policies:

$$\mathrm{Circ}(\mathcal{T}_{\mathrm{psl}} \cup \mathcal{T}_{\mathrm{dt}}, ; subact\_occ; min\_prec, root\_occ, leaf\_occ, next\_subocc) \wedge \mathrm{Circ}(\mathcal{T}_{\mathrm{psl}} \cup \mathcal{T}_{\mathrm{dt}}, ; extact\_order; subact\_occ, min\_prec, next\_subocc, leaf\_occ) \quad (45)$$

The models of the theory that contains unnecessary subactivity occurrences and unnecessary external occurrences will be eliminated in the models of the combined circumscription policies.

Discussion

In this paper, we proposed formal characterizations of the intended models of process descriptions and provided circumscription based minimization policies to allow reasoning within those models. First we characterized a closed interpretation of process descriptions that only specify the subactivities that occur and constraints on their occurrences. Intuitively, the closed models of such process descriptions correspond to how we naturally interpret 'partial' task descriptions, say a recipe for a dish. In other words, when carrying out a complex activity based on its description, only the explicitly specified actions are understood to be performed as many times as they are required to be performed by the description.

Reasoning with complex activities require determining how their occurrences affect the world. Closing process descriptions of complex activities addresses the problem of determining which activities occur as part of their occurrences, however, in general it is consistent that various other external activity occurrences interleave with their occurrences. We provided a closure assumption on external activities that allow reasoning about the complex activities with minimal interference from the external world.

On the minimization of the process descriptions side, our intention is to explore if the minimal models can be obtained within the first order logic without the use of a circumscriptive technique in the future. PSL allows reasoning about consistent occurrences of an activity within a model. The need to compare models can be eliminated if a very large model can be constructed that contains all the consistent occurrences. Currently we do not know whether such maximal models are first-order definable. Alternatively, a procedure can be defined to compile the closed descriptions from the partial ones, like Ray Reiter's approach to generate successor state axioms from effect axioms(Reiter 1991). However, such procedure would require a particular syntactic form for the specification of process descriptions. Not only this syntactic form should be expressive enough to specify all possible members of all classes of activities, but also it should be natural to write them in this form. Otherwise it would be preferable to write them without the syntactic restrictions and provide explicit closure axioms. In (Pinto 1994), Pinto provided a Prolog implementation of his framework that preserved the reasoning properties of the circumscribed models. We will also explore the reasoning properties of partially specified process descriptions within a disjunctive logic programming framework.

Minimization of external activities within first-order logic may also be possible in a framework like PSL. We have shown in the last section that preferred occurrences within a model can be described in first order (the *minimal* relation). Then the problem reduces to making sure every model contains minimal occurrences. However this property of the models may not be first-order definable as we stated earlier.

The minimization policy for the external activities seems quite natural, however, there can be alternative intuitions as to which models should be considered minimal. In Figure 7, it can be argued that the occurrences must be comparable and that the occurrence in $\mathcal{M}^2$ is more minimal. Our minimization policy takes the conservative approach and accepts both occurrences to be minimal. Process descriptions consistent with such boundary models must be empirically studied to gain more concrete insight into reasonable conclusions in theses cases.

The related work in the literature studies the problem of reasoning about complex actions under considerably more restricted frameworks. In (McIlraith and Fadel 2002), the authors provide a mechanism to compile complex actions specified as procedures in the Golog language(Levesque et al. 1997) into primitive actions. Although, complex actions and their occurrences are not objects in Golog, turning them into primitive actions makes it possible to reason about them within the language. However, Golog does not incorporate exogenous actions which severely restricts the validity of the conclusions reached by reasoning about complex actions. Similar work in (Grüninger and Pinto 1995) uses a framework where complex actions are objects in the language but not their occurrences and models only represent actual occurrences not the hypothetically possible ones. The minimization policy presented in (Grüninger and Pinto 1995) not only intrinsically limited due to the underlying framework but it also does not guarantee to consider all the intended models when reasoning.

To our best knowledge, the work we presented in this paper uses the most general setting for studying the problem of reasoning about complex actions.

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